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Hydrodynamics and heat transfer of turbulent gas suspension flows in tubes—2. Heat transfer

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Abstract—By the method of averaging over the ensemble of turbulent flow realizations, averaged heat transfer equations for a solid phase and a flow as a whole are derived. Closed expressions for the second single-point moments of the solid and carrier phase velocity and temperature fluctuations in terms of the second moments of the carrier phase velocity and temperature fluctuations in a non-uniform turbulent flow are found. Based on these expressions, a set of equations is written for the second single-point moments of the liquid phase velocity and temperature fluctuations in the presence of particles. Heat transfer calculations are carried out for turbulent flow of gas suspension in circular tubes. The effect of the relationship between the thermal and physical properties of the particle material and gas on the thermal characteristics of a two-phase flow is investigated. The predicted Nusselt numbers for a dusty flow agree satisfactorily with the experimental data.

1. INTRODUCTION

THIS PAPER is a continuation of the previous paper [1] and is based on its results. The forthcoming review of theoretical studies dealing with heat transfer of turbulent disperse flows makes no claims to completeness; its aim is to illustrate the basic methods which are employed for calculating heat transfer of such two-phase flows.

In refs. [2, 3] an analysis of the heat transfer of a disperse flow was carried out based on a two-layer model of turbulent transfer and on an empirical relation for the turbulent viscosity coefficient of a disperse flow. To calculate the coefficient of heat exchange between the flow and the wall, additional information on the hydraulic resistance coefficient is required, and this poses a self-sustained problem. The results of calculations made in ref. [4] and based on the van Driest damping model, modified to take into account the effect of admixture on turbulence, correlate well with experimental data and adequately represent the nature of the dependence of heat transfer on the concentration and size of particles. However, the solution obtained in that work contains some additional constants associated with the presence of particles and determined by comparing with experimental data. The hypothesis about the suppression by particles of turbulent fluctuations the characteristic size of which is smaller than the particle diameter [5] was taken as a basis for the model of heat transfer in gas suspension with relatively large particles [6]. Calculations based on the latter model also employ additional constants the values of which are selected by comparing prediction with experiment. References [2–4, 6] primarily study the effect of the size, material density and mass concentration of admixture particles on the heat transfer of gas suspension flows hardly touching on the problem of the effect which is exerted by the relationship between the thermophysical properties of the discrete and liquid phases of heat transfer of disperse flows. This factor is of interest in regard to the problem of controlling the intensity of heat transfer of dusty flows; thus, for example, an increase in the heat capacity ratio of the particle material and gas can significantly enhance heat transfer of a disperse flow [7, 8]. Also, there is virtually no investigations into the effect of the carrier phase Prandtl number on transfer of a turbulent disperse flow.

In the present paper, using the method of averaging over the ensemble of turbulent flow realizations, equations are obtained which describe turbulent heat transfer in a solid phase and in a dusty flow as a whole. Taking into account the turbulent flow inhomogeneity, closed expressions were found for correlating the velocity and temperature fluctuations of the discrete and liquid phases. With these expressions being incorporated in the equations for the second single-point moments of the carrier phase velocity and temperature fluctuations, calculations were carried out for heat transfer of a gas suspension flow in a circular tube. As a result, the effect of the relative size and mass concentration of particles and of the ratio between the thermophysical properties of the discrete and liquid phases on the thermal characteristics of a disperse turbulent flow was studied.

2. AVERAGED HEAT TRANSFER EQUATIONS FOR A DISCRETE PHASE AND A FLOW AS A WHOLE

A theoretical examination is made of the heat transfer of a disperse flow of small particles the dynamic relaxation time of which is commensurable with the

NOMENCLATURE

a particle radius
$\langle C \rangle$ volume concentration of solid phase
c_1, c_2 heat capacity of particle material and gas, respectively
D_K^ϑ 'thermal' coefficient of diffusion
E turbulent energy of carrier phase
$F_\Theta(s), F_{\Theta u}(s)$ two-time correlation functions of carrier phase temperature and velocity fluctuations
$f_{u2}, f_{u3}, g_{\vartheta u1}, g_{u4}, g_{\vartheta 4}$ coefficients describing the degree of entrainment of particles into pulsating motion
L spatial scale of turbulence
m ratio of thermal to dynamic relaxation times of particles, τ_ϑ/τ_u
N number of particles per flow volume
Nu, Nu_0 Nusselt number of gas suspension flow and single-phase flow, respectively
P pressure fluctuations in carrier phase
Pr_1, Pr_t molecular and turbulent Prandtl number of gas, respectively
q_w heat flux at the tube wall
R tube radius
$R_p(t)$ radius vector of pth particle
Re main stream Reynolds number
Re_E turbulent Reynolds number
T_E time scale of turbulence
T_0 characteristic time of variation of averaged quantities
$\langle U_i \rangle, u_i$ averaged and pulsating velocities of carrier phase
U_m average velocity of carrier phase
$\langle V_i \rangle, v_i$ averaged and pulsating velocities of discrete phase
$V_{pk}(t)$ velocity of pth particle
x, x_k Cartesian coordinates
x, r longitudinal and radial coordinates
$\tilde{y}$ distance from the tube wall normalized to the tube radius.

Greek symbols
$\delta(x)$ Dirac delta function
$\varepsilon_i^{\vartheta u}, \varepsilon^\vartheta$ terms describing interphase interaction in the equations of second single-point moments for velocity and temperature fluctuations of particle-laden gas
$\Theta_1, \langle \Theta_1 \rangle, \vartheta_1$ actual, averaged and fluctuating temperatures of the liquid phase
$\Theta_2, \langle \Theta_2 \rangle, \vartheta_2$ actual, averaged and fluctuating temperatures of the discrete phase
Θ_w, Θ_m wall and bulk temperatures
ν_1, ν_{t1} coefficient of molecular and turbulent kinematic viscosities of the liquid phase
κ_1 thermal diffusivity of gas
κ_{t1}, κ_{t2} turbulent thermal diffusivity of the liquid and solid phase, respectively
ρ_2, ρ_1 density of particle material and gas, respectively
τ_u time of dynamic relaxation of particles, $(2/9)(\rho_2/\rho_1)(a^2/\nu_1)$
τ_ϑ time of thermal relaxation of particles, $(3/2)(\rho_2 c_2/\rho_1 c_1)(a^2/\kappa_1)$
Ω_u coefficient of dynamic relaxation of particles
Ω_ϑ coefficient of thermal relaxation of particles.

lifetime of energy-carrying fluctuations of the carrier phase. It is assumed that the thermophysical properties of the solid and carrier phases are constant, whereas the thermal conductivity of the particle material substantially exceeds that of the carrier phase. Phenomena associated with the particle thermophoresis are not touched upon. Variation in the temperature of a single particle and of the carrier phase is described by the equations

$$\frac{d\Theta_p}{dt} = \frac{1}{\tau_\Theta}(\Theta_1(R_p(t), t) - \Theta_p(t)) \tag{1}$$

$$\frac{\partial \Theta_1}{\partial t} + U_k \frac{\partial \Theta_1}{\partial x_k} = \kappa_1 \frac{\partial^2 \Theta_1}{\partial x_k \partial x_k} - \frac{\rho_2 c_2}{\rho_1 c_1} \sum_{p=1}^{N} \delta(x - R_p(t)) \frac{d\Theta_p(t)}{dt}. \tag{2}$$

The distribution of the solid phase temperature throughout the flow volume is represented as

$$\Theta_2(x,t)C(x,t) = \frac{\omega}{\Omega_N} \sum_{p=1}^{N} \delta(x - R_p(t))\Theta_p(t).$$

Averaging over the ensemble of turbulent flow realizations, isolate the averaged and fluctuating components of the liquid and solid phase temperatures

$$\Theta_1(x,t) = \langle \Theta_1(x,t) \rangle + \Theta_1(x,t), \quad \langle \vartheta_1(x,t) \rangle = 0$$

$$\Theta_2(x,t) = \langle \Theta_2(x,t) \rangle + \vartheta_2(x,t)$$

where

$$\langle C(x,t) \rangle \langle \Theta_2(x,t) \rangle = \left\langle \frac{\omega}{\Omega_N} \sum_{p=1}^{N} \delta(x - R_p(t))\Theta_p(t) \right\rangle \tag{3}$$

$$\langle C(x,t)\vartheta_2(x,t) \rangle = I. \tag{4}$$

Differentiating equation (3) with respect to time, with equations (1) and (4) taken into account, the equations for the solid phase averaged temperature is obtained

$$\langle C\rangle\left(\frac{\partial\langle\Theta_2\rangle}{\partial t}+\langle V_k\rangle\frac{\partial\langle\Theta_2\rangle}{\partial x_k}\right) = -\langle C\rangle\frac{\partial\langle\vartheta_2 v_k\rangle}{\partial x_k}-\langle\vartheta_2 v_k\rangle\frac{\partial\langle C\rangle}{\partial x_k}+\frac{1}{\tau_\vartheta}\langle C\vartheta_1\rangle + \frac{\langle C\rangle}{\tau_\vartheta}(\langle\Theta_1\rangle-\langle\Theta_2\rangle). \quad (5)$$

It is seen from equation (5) that variations in the solid phase temperature are attributable to a turbulent flow of heat in the solid phase because of the entrainment of particles into pulsating motion [the first term on the right-hand side of equation (5)], to the diffusional transfer of temperature fluctuations [the second term on the right-hand side of equation (5)], and also to the interphase heat transfer [the last two terms on the right-hand side of equation (5)].

Having averaged equation (2) and added it to equation (5), the heat balance equation for the flow as a whole can be written as

$$\frac{\partial\langle\Theta_1\rangle}{\partial t}+\langle U_k\rangle\frac{\partial\langle\Theta_1\rangle}{\partial x_k}+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle\left(\frac{\partial\langle\Theta_2\rangle}{\partial t}+\langle V_k\rangle\frac{\partial\langle\Theta_2\rangle}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\left[\kappa_1\frac{\partial\langle\Theta_1\rangle}{\partial x_k}-\left(\langle\vartheta_1 u_k\rangle+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle\langle\vartheta_2 v_k\rangle\right)\right]. \quad (6)$$

It follows from the above equation that the admixture particles not only contribute to convective heat transfer, but also enhance the turbulent heat flow due to their entrainment into pulsating motion.

3. EQUATIONS FOR THE SECOND MOMENTS OF THE PARTICLE-LADEN CARRIER PHASE VELOCITY AND TEMPERATURE FLUCTUATIONS

Equations for the second single-point moments of the velocity and temperature fluctuations of the carrier phase laden with particles result from equation (2) and coincide with the corresponding equations for a single-phase flow, except for the additional terms arising due to the interphase interaction. The additional term in the equation for the second single-point moments of the liquid phase velocity and temperature fluctuations has the following form:

$$\varepsilon_i^{\vartheta u}=\frac{\rho_2}{\rho_1}\left\langle\frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))\left[\frac{c_2}{c_1}\frac{\mathrm{d}\Theta_p(x,t)}{\mathrm{d}t}\times u_i(x,t)+\frac{\mathrm{d}V_{pi}(t)}{\mathrm{d}t}\vartheta_1(x,t)\right]\right\rangle. \quad (7)$$

The additional interphase heat transfer term in the equation for the second single-point moments of the carrier phase temperature fluctuations is

$$\varepsilon^{\vartheta}=2\frac{\rho_2}{\rho_1}\left\langle\frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))\frac{\mathrm{d}\Theta_p(t)}{\mathrm{d}t}\vartheta_1(x,t)\right\rangle. \quad (8)$$

Substituting the single particle motion and heat transfer equations into equations (7) and (8) gives

$$\varepsilon_i^{\vartheta u}=\frac{\rho_2}{\rho_1}\frac{\langle C\rangle}{\tau_u}(\langle\vartheta_1 u_i\rangle-\langle\vartheta_1 v_i\rangle)+\frac{\langle C\vartheta_1\rangle}{\tau_u}(\langle U_i\rangle-\langle V_i\rangle)+\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\langle C\rangle}{\tau_\vartheta}(\langle\vartheta_1 u_i\rangle-\langle\vartheta_2 u_i\rangle)+\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\langle C u_i\rangle}{\tau_\vartheta}(\langle\Theta_1\rangle-\langle\Theta_2\rangle) \quad (9)$$

$$\varepsilon^{\vartheta}=2\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\langle C\rangle}{\tau_\vartheta}(\langle\vartheta_1^2\rangle-\langle\vartheta_1\vartheta_2\rangle)+2\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\langle C\vartheta_1\rangle}{\tau_\vartheta}(\langle\Theta_1\rangle-\langle\Theta_2\rangle). \quad (10)$$

Equation (5) for the solid phase averaged temperature and the expressions for $\varepsilon_i^{\vartheta u}$ and ε^{ϑ} include single-point correlation moments of the solid and carrier phase velocity and temperature fluctuations. For these to be calculated, assume that the solid phase temperature fluctuations are caused by the fluctuating temperature slip of phases, i.e.

$$\frac{\partial\vartheta_2}{\partial t}+\langle V_k\rangle\frac{\partial\vartheta_2}{\partial x_k}+V_k\frac{\partial\langle\Theta_2\rangle}{\partial x_k}+\frac{\partial(\vartheta_2 v_k-\langle\vartheta_2 v_k\rangle)}{\partial x_k}+\frac{1}{\tau_\vartheta}(\vartheta_1-\vartheta_2). \quad (11)$$

Writing down equation (11) in integral form, multiplying the result by the carrier phase velocity or temperature fluctuations and averaging, obtain closed expressions for the correlations of temperature and velocity fluctuations of the solid and carrier phase accurate to the terms of the order of $(T_E/T_0)^2$:

$$\langle\vartheta_2 u_i\rangle=g_{\vartheta 1}\langle\vartheta_1 u_i\rangle-\tau_\vartheta g_{\vartheta 2}\left[\frac{1}{2}\frac{\partial\langle\vartheta_1 u_i\rangle}{\partial t}+\langle V_k\rangle\left\langle u_i\frac{\partial\vartheta_1}{\partial x_k}\right\rangle+\frac{g_{\vartheta 2}}{g_{\vartheta 2}}\langle u_i u_k\rangle\times\frac{\partial\langle\Theta_2\rangle}{\partial x_k}+\left\langle u_i\frac{\partial\vartheta_1 u_k}{\partial x_k}\right\rangle\right] \quad (12)$$

$$\langle\vartheta_1 v_k\rangle=g_{u1}\langle\vartheta_1 u_i\rangle-\tau_u g_{u2}\left[\frac{1}{2}\frac{\partial\langle\vartheta_1 u_i\rangle}{\partial t}+\langle V_k\rangle\left\langle\vartheta_1\frac{\partial u_i}{\partial x_k}\right\rangle+\frac{g_{\vartheta 2}}{g_{u2}}\langle\vartheta_1 u_k\rangle\times\frac{\partial\langle V_i\rangle}{\partial x_k}+\left\langle\vartheta_1\frac{\partial u_i u_k}{\partial x_k}\right\rangle\right] \quad (13)$$

$$\langle \vartheta_1\vartheta_2\rangle = f_{\vartheta 1}\langle \vartheta_1^2\rangle - \tau_\vartheta f_{\vartheta 2}\left[\frac{1}{2}\frac{\partial\langle \vartheta_1^2\rangle}{\partial t} + \frac{1}{2}\langle V_k\rangle\frac{\partial\langle \vartheta_1^2\rangle}{\partial x_k} + \frac{g_{\vartheta u2}}{f_{\vartheta 2}}\langle \vartheta_1 u_k\rangle\frac{\partial\langle \Theta_2\rangle}{\partial x_k} + \frac{1}{2}\frac{\partial\langle \vartheta_1^2 u_k\rangle}{\partial x_k}\right]. \quad (14)$$

Analogously, using the method of averaging over the ensemble of turbulent flow realizations for the equation of temperature fluctuations (11), obtain expressions for the correlations accurate to the terms of the order of

$$\langle \vartheta_2 v_i\rangle = g_{\vartheta u1}\langle \vartheta_1 u_i\rangle - \frac{\tau_u\tau_\vartheta}{\tau_u+\tau_\vartheta}(g_{\vartheta u2}+g_{\vartheta u1})\left[\frac{\partial\langle \vartheta_1 u_i\rangle}{\partial t} + \langle V_k\rangle\frac{\partial\langle \Theta_1 u_i\rangle}{\partial x_k} + \langle \vartheta_1 u_k\rangle\frac{\partial\langle V_i\rangle}{\partial x_k} + \langle u_i u_k\rangle\frac{\partial\langle \Theta_2\rangle}{\partial x_k} + \frac{\partial\langle \vartheta_1 u_i u_k\rangle}{\partial x_k}\right] \quad (15)$$

$$\langle \vartheta_2^2\rangle = f_{\vartheta 1}\langle \vartheta_1^2\rangle - \tfrac{1}{2}\tau_\vartheta(f_{\vartheta 2}+f_{\vartheta 1})\left[\frac{\partial\langle \vartheta_1^2\rangle}{\partial t} + \langle V_k\rangle\frac{\partial\langle \vartheta_1^2\rangle}{\partial x_k} + 2\frac{g_{\vartheta u2}+g_{\vartheta u1}}{f_{\vartheta 2}+f_{\vartheta 1}}\langle \vartheta_1 u_k\rangle \times\frac{\partial\langle \Theta_2\rangle}{\partial x_k} + \frac{\partial\langle \vartheta_1^2 u_k\rangle}{\partial x_k}\right]. \quad (16)$$

The functions f, g, q in equations (12)–(16) describe the degree with which the admixture particles are entrained into the carrier phase temperature fluctuations. They have the following form:

$$f_{\vartheta 1} = \frac{1}{\tau_\vartheta}\int_0^\infty \exp\left(-\frac{s}{\tau_\vartheta}\right)F_\vartheta(s)\,ds$$

$$f_{\vartheta 2} = \frac{1}{\tau_\vartheta^2}\int_0^\infty s\exp\left(-\frac{s}{\tau_\vartheta}\right)F_\vartheta(s)\,ds$$

$$g_{\vartheta u1} = \frac{1}{\tau_u+\tau_\vartheta}\int_0^\infty\left[\exp\left(-\frac{s}{\tau_u}\right) + \exp\left(-\frac{s}{\tau_\vartheta}\right)\right]F_{\vartheta u}(s)\,ds$$

$$g_{u2} = \frac{1}{\tau_u^2}\int_0^\infty s\exp\left(-\frac{s}{\tau_u}\right)F_{\vartheta u}(s)\,ds$$

$$g_{\vartheta 2} = \frac{1}{\tau_\vartheta^2}\int_0^\infty s\exp\left(-\frac{s}{\tau_\vartheta}\right)F_{\vartheta u}(s)\,ds$$

$$g_{\vartheta u2} = \frac{1}{\tau_\vartheta-\tau_u}\int_0^\infty\left[\exp\left(-\frac{s}{\tau_\vartheta}\right) - \exp\left(-\frac{s}{\tau_u}\right)\right]F_{\vartheta u}(s)\,ds$$

$$g_{\vartheta 1} = \frac{1}{\tau_\vartheta}\int_0^\infty \exp\left(-\frac{s}{\tau_\vartheta}\right)F_{\vartheta u}(s)\,ds$$

$$g_{u1} = \frac{1}{\tau_u}\int_0^\infty \exp\left(-\frac{s}{\tau_u}\right)F_{\vartheta u}(s)\,ds$$

$$q_{u2} = \frac{1}{\tau_\vartheta-\tau_u}\int_0^\infty\left[\exp\left(-\frac{s}{\tau_\vartheta}\right) - \exp\left(-\frac{s}{\tau_u}\right)\right]F_u(s)\,ds$$

$$q_{\vartheta 2} = \frac{1}{\tau_\vartheta-\tau_u}\int_0^\infty\left[\exp\left(-\frac{s}{\tau_\vartheta}\right) - \exp\left(-\frac{s}{\tau_u}\right)\right]F_\vartheta(s)\,ds. \quad (17)$$

The functions $F_{\vartheta u}(s)$ and $F_\vartheta(s)$ determine two-time correlation moments of the carrier phase velocity and temperature fluctuations

$$\langle \vartheta_1(x,t)u_i(x,t+s)\rangle = F_{\vartheta u}(s)\langle \vartheta_1(x,t)u_i(x,t)\rangle$$
$$\langle \vartheta_1(x,t)\vartheta_1(x,t+s)\rangle = F_\vartheta(s)\langle \vartheta_1^2(x,t)\rangle.$$

By assigning the specific form of the functions $F_{\vartheta u}(s)$ and $F_\vartheta(s)$, it is possible to obtain from equations (12)–(17), closed expressions for the correlations of the solid and carrier phase velocity and temperature fluctuations in terms of the second single-point moments of liquid phase velocity and temperature fluctuations taking account of the nonuniformity and unsteadiness of the turbulent flow. In this case, the first terms on the right-hand sides of equations (12)–(16) represent correlations of the solid and carrier phase velocity and temperature fluctuations in a uniform steady-state turbulent flow, whereas the terms within square brackets describe the contribution of the unsteadiness and nonuniformity of the carrier phase velocity and temperature fluctuation.

For particles with low dynamic and thermal inertia ($\tau_u \to 0$, $\tau_\vartheta \to 0$), obtain $\langle \vartheta_2 u_i\rangle = \langle \vartheta_1 v_1\rangle = \langle \vartheta_1 u_i\rangle$, $\langle \vartheta_1\vartheta_2\rangle = \langle \vartheta_2^2\rangle = \langle \vartheta_1^2\rangle$; for inertia particles the dynamic and thermal relaxation times of which considerably exceed the lifetime of thermal-carrying moles, obtain $\langle \vartheta_2 u_i\rangle \to \langle \vartheta_1 v_i\rangle \to \langle \vartheta_1\vartheta_2\rangle \to 0$, but due to the additional generation of the intensity of solid phase fluctuations in a non-uniform turbulent flow $\langle \vartheta_2 v_i\rangle = O(\tau_u/T_0)$, $\langle \vartheta_2^2\rangle = O(\tau_\vartheta/T_0)$.

The expression for the correlation $\langle C\vartheta_1\rangle$ is found similarly to the expression for $\langle Cu_i\rangle$ in ref. [1]. It has the following form:

$$\langle C(x,t)\vartheta_1(x,t)\rangle = -\tau_u g_{\vartheta u3}\langle \vartheta_1 u_k\rangle\frac{\partial\langle C\rangle}{\partial x_k} \quad (18)$$

where

$$g_{\vartheta u3} = \frac{1}{\tau_u}\int_0^\infty\left[1-\exp\left(-\frac{s}{\tau_u}\right)\right]F_{\vartheta u}(s)\,ds.$$

Using equation (18) the solid phase heat transfer equation (6) can be written as

$$\frac{\partial\langle\Theta_2\rangle}{\partial t}+\langle V_k\rangle\frac{\partial\langle\Theta_2\rangle}{\partial x_k}=-\frac{\partial\langle\vartheta_2 v_k\rangle}{\partial x_k}+\frac{1}{\tau_\vartheta}(\langle\Theta_1\rangle-\langle\Theta_2\rangle)-\frac{1}{\tau_\vartheta}D_k^\vartheta\frac{\partial\ln\langle C\rangle}{\partial x_k} \quad (19)$$

where

$$D_k^\vartheta=(\tau_\vartheta g_{\vartheta u1}+\tau_u g_{\vartheta u3})\langle\vartheta_1 u_k\rangle=\int_0^1\left[1-\frac{\tau_u\exp\left(-\frac{s}{\tau_u}\right)-\tau_\vartheta\exp\left(-\frac{s}{\tau_\Theta}\right)}{\tau_u+\tau_\vartheta}\right]\times F_{\vartheta u}(s)\,\mathrm{d}s$$

is the 'thermal' coefficient of diffusion.

Equations (7)–(10), (12)–(16) and (18) yield a set of equations for the second single-point moments of the velocity and temperature fluctuations and also for the square of the particle-laden carrier phase temperature fluctuations

$$\left(1+\frac{\rho_2}{\rho_1}\langle C\rangle g_{u2}+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle g_{\vartheta 2}\right)\frac{\partial\langle\vartheta_1 u_i\rangle}{\partial t}+\langle U_k\rangle\frac{\partial\langle\vartheta_1 u_k\rangle}{\partial x_k}+\frac{\rho_2}{\rho_1}\langle C\rangle\langle V_k\rangle\left(\left\langle\vartheta_1\frac{\partial u_i}{\partial x_k}\right\rangle g_{u2}+\frac{c_2}{c_1}\left\langle u_i\frac{\partial\vartheta_1}{\partial x_k}\right\rangle g_{\vartheta 2}\right)+\langle\vartheta_1 u_k\rangle\left(\frac{\partial\langle U_i\rangle}{\partial x_k}+\frac{\rho_2}{\rho_1}\langle C\rangle g_{\vartheta 2}\frac{\partial\langle V_i\rangle}{\partial x_k}\right)+\frac{\partial\langle\vartheta_1 u_i u_k\rangle}{\partial x_k}$$
$$+\frac{\rho_2}{\rho_1}\langle C\rangle\left[\left\langle\vartheta_1\frac{\partial u_i u_k}{\partial x_k}\right\rangle g_{u2}+\frac{c_2}{c_1}\left\langle u_i\frac{\partial\vartheta_1}{\partial x_k}\right\rangle g_{\vartheta 2}\right]=-\frac{1}{\rho_1}\left\langle\vartheta_1\frac{\partial P}{\partial x_i}\right\rangle+\nu_1\left\langle\vartheta_1\frac{\partial^2 u_i}{\partial x_k\partial x_k}\right\rangle+\kappa_1\left\langle u_i\frac{\partial^2\vartheta_1}{\partial x_k\partial x_k}\right\rangle$$
$$-\frac{\rho_2}{\rho_1}g_{\vartheta u3}\langle\vartheta_1 u_k\rangle\frac{\partial\langle C\rangle}{\partial x_k}(\langle U_i\rangle-\langle V_i\rangle)-\frac{\tau_u}{\tau_\vartheta}\frac{\rho_2 c_2}{\rho_1 c_1}f_{u3}\frac{\partial\langle C\rangle}{\partial x_k}(\langle\Theta_1\rangle-\langle\Theta_2\rangle)-\frac{\rho_2}{\rho_1}\frac{\langle C\rangle}{T_E}\left(g_{u5}+\frac{c_2}{c_1}g_{\vartheta 5}\right)\langle\vartheta_1 u_i\rangle \quad (20)$$

$$\left(1+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle g_{\vartheta 2}\right)\frac{\partial\langle\Theta_1^2\rangle}{\partial t}+\left(\langle U_k\rangle+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle f_{\vartheta 2}\langle V_k\rangle\right)\frac{\partial\langle\vartheta_1^2\rangle}{\partial x_k}+2\langle\vartheta_1 u_k\rangle\left(\frac{\partial\langle\Theta_1\rangle}{\partial x_k}+\frac{\rho_2 c_2}{\rho_1 c_1}\langle C\rangle g_{\vartheta u2}\frac{\partial\langle\Theta_2\rangle}{\partial x_k}\right)+\left(1+\frac{c_2\rho_2}{c_1\rho_1}\langle C\rangle g_{\vartheta u2}\right)\frac{\partial\langle\vartheta_1^2 u_k\rangle}{\partial x_k}$$
$$=\kappa_1\frac{\partial^2\vartheta_1^2}{\partial x_k\partial x_k}-2\kappa_1\left\langle\frac{\partial\vartheta_1}{\partial x_k}\frac{\partial\vartheta_1}{\partial x_k}\right\rangle-2\frac{\rho_2 c_2}{\rho_1 c_1}g_{\vartheta u3}\langle\vartheta_1 u_k\rangle\frac{\partial\langle C\rangle}{\partial x_k}(\langle\Theta_1\rangle-\langle\Theta_2\rangle)-2\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\langle C\rangle}{T_E}f_{\vartheta 5}\langle\vartheta_1^2\rangle \quad (21)$$

where

$$g_{u4}=\frac{T_E}{\tau_u}(1-g_{u1}),\quad g_{\vartheta 4}=\frac{T_E}{\tau_u}(1-g_{\vartheta 1}),$$
$$f_{\vartheta 4}=\frac{T_E}{\tau_\vartheta}(1-f_{\vartheta 1}).$$

It is seen from equations (20) and (21), that particles, being entrained into the pulsating motion of the carrier phase, contribute to the terms that describe convective transfer, turbulent diffusion of the carrier phase temperature fluctuations and fluctuation origination from averaged motion. Moreover, new terms appear that describe the variation in temperature fluctuations due to the admixture concentration gradient [the last but one terms in equations (20) and (21)] and the additional dissipation of the carrier phase fluctuations by particles [the last terms in equations (20) and (21)].

For inertia-free particles ($\tau_u\to 0$, $\tau_\vartheta\to 0$, $g_{u2}\to 1$, $g_{\vartheta 2}\to 1$, $g_{\vartheta u2}\to 1$, $g_{\vartheta u4}\to 0$, $g_{\vartheta 4}\to 0$, $f_{\vartheta 4}\to 0$), the convective and turbulent diffusion transfer and also the origination of temperature fluctuations increase in equations (20) and (21) as compared with the single-phase liquid flow. Inertia particles ($\tau_u\sim T_E$) are less entrained into pulsating motion ($g_{u2}<1$, $g_{\vartheta 2}<1$, $g_{\vartheta u2}<1$, $g_{u4}\neq 0$, $g_{\vartheta 4}\neq 0$, $f_{\vartheta 4}\neq 0$) and can lead to a decrease in the intensity of temperature fluctuations. Large particles ($\tau_u\gg T_E$, $\tau_\vartheta\gg T_E$) are not entrained into the carrier phase temperature fluctuations and therefore $g_{u2}, g_{\vartheta 2}, g_{\vartheta u2}, g_{\vartheta 4}, g_{u4}, f_{\vartheta 4}\to 0$.

4. EQUATIONS FOR CALCULATING HEAT TRANSFER OF TURBULENT FLOWS IN TUBES

As with the calculation of hydrodynamics [1], the two-time correlation functions of the carrier phase velocity and temperature fluctuations are given in the form

$$F_{\vartheta u}(s)=F_\vartheta(s)=\begin{cases}1, & \text{when } 0\leqslant s\leqslant T_E\\ 2, & \text{when } s>T_E\end{cases}.$$

Substituting equation (24) into equations (16) and (23), the following expressions for the coefficients that

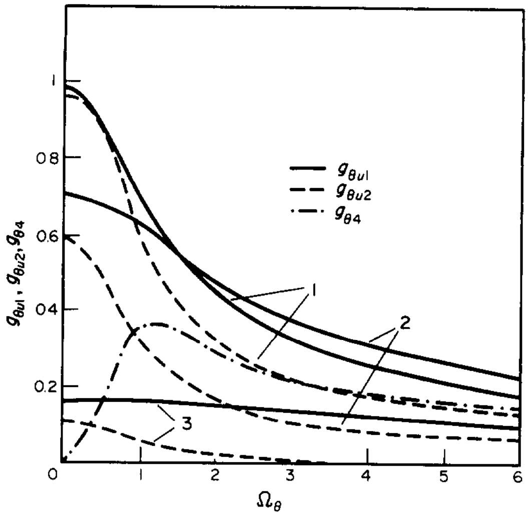

FIG. 1. Coefficients representing the contribution of particles into averaged heat transfer, additional generation and dissipation of gas temperature fluctuations 1, $\Omega_u = 0.2$, 1, 10.

describe the degree of the entrainment of the particles into the gas temperature fluctuations are obtained:

$$g_{u1} = 1-\exp(-1/\Omega_u),\ f_{\vartheta 1} = g_{\vartheta 1} = 1-\exp(-1/\Omega_\vartheta)$$
$$g_{u2} = 1-(1+1/\Omega_u)\exp(-1/\Omega_u)$$
$$f_{\vartheta 2} = g_{\vartheta 2} = 1-(1+1/\Omega_\vartheta)\exp(-1/\Omega_\vartheta)$$
$$g_{\vartheta u1} = 1-[\Omega_\vartheta \exp(-1/\Omega_\vartheta) + \Omega_u \exp(-1/\Omega_u)/(\Omega_\vartheta+\Omega_u)$$
$$g_{u2} = q_{\vartheta 2} = q_{u2} = 1-[\Omega_\vartheta \exp(-1/\Omega_\vartheta) - \Omega_u \exp(-1/\Omega_u)/(\Omega_\vartheta-\Omega_u)$$
$$g_{u4} = \exp(-1/\Omega_u)/\Omega_u$$
$$g_{04} = f_{04} = \exp(-1/\Omega_\vartheta)/\Omega_\vartheta. \quad (22)$$

Formulae (22) include the thermal inertia parameter $\Omega_\vartheta = \tau_\vartheta/T_E$ which is analogous in meaning to the parameter of the dynamic inertia of particles $\Omega_u = \tau_u/T_E$.

Figure 1 illustrates the dependence of the coefficients that describe the entrainment of particles into temperature fluctuations on the particle thermal inertia parameter. It is seen that as the thermal inertia parameter increases, the concentration of particles into turbulent heat transfer diminishes. The maximum of the additional dissipation of temperature fluctuations on particles is reached at $\Omega_\vartheta = 1$.

To describe the dissipation and exchange terms in equations (20) and (21), use is made of the Monin–Kolovandin approximation hypotheses [9, 10]

$$\kappa_1\left\langle\left(\frac{\partial \vartheta_1}{\partial x_k}\right)^2\right\rangle = c_\vartheta \frac{E^{1/2}}{L}\langle\vartheta_1^2\rangle + c_{1\vartheta}\frac{\langle\vartheta_1^2\rangle}{L^2}$$
$$-\left\langle\frac{P}{\rho_1}\frac{\partial\vartheta_1}{\partial x_k}\right\rangle = k_\vartheta \frac{E^{1/2}}{L}\langle\vartheta_1 u_i\rangle. \quad (23)$$

In ref. [11], a detailed analysis of the effect of particles on the intensity of turbulent heat transfer of a gas suspension is performed. A diffusion-free approximation for large turbulent Reynolds numbers, $Re_E \gg 1$, gives an algebraic set of equations for the second single-point moments of velocity and temperature fluctuations of a carrying gas with particles. The solution of this set of equations yields in the form of modified Prandtl ratios that determine the intensity of temperature fluctuations of a turbulent heat flow and also expressions that define the turbulent Prandtl number for the carrying gas. Analysis of the resulting expressions shows that particles with the dynamic inertia parameter exceeding unity weaken the intensity of temperature fluctuations and the turbulent heat flow of the carrying gas. Particles with low dynamic and thermal inertia parameters intensify the processes of turbulent transfer, with the growth of the particle thermal inertia decreasing the intensity of the carrier phase temperature fluctuations.

Heat transfer for a hydrodynamically and thermally stabilized turbulent disperse flow in a circular tube will now be calculated. In this case the averaged velocities and temperatures of both phases can be assumed equal, and the heat transfer equation (6) for the disperse flow in the boundary layer theory approximation can be stated as follows:

$$\left(1+\frac{c_2}{c_1}\langle\Phi\rangle\right)\left(\langle U_x\rangle\frac{\partial\langle\Theta_1\rangle}{\partial x} + \langle U_r\rangle\frac{\partial\langle\Theta_1\rangle}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\kappa_1\frac{\partial\langle\Theta_1\rangle}{\partial r} - \langle\vartheta_1 u_r\rangle - \frac{c_2}{c_1}\langle\Phi\rangle\langle\vartheta_2 v_r\rangle\right)\right]. \quad (24)$$

Turbulent flows of heat in the liquid and solid phases are represented by the coefficients of turbulent thermal diffusivity as

$$\langle\vartheta_1 u_r\rangle = -\kappa_{t1}\frac{\partial\langle\Theta_1\rangle}{\partial r} = -\frac{\nu_{t1}}{Pr_t}\frac{\partial\langle\Theta_1\rangle}{\partial r}$$
$$\langle\vartheta_2 v_r\rangle = -\kappa_{t2}\frac{\partial\langle\Theta_1\rangle}{\partial r}.$$

The relationship between the turbulent thermal diffusivities of the solid and carrier phases in a non-uniform turbulent flow is determined from formula (12) and has the form

$$\kappa_{t2} = \kappa_{t1} g_{\vartheta u1}\left\{1+\frac{m}{m+1}\frac{\Omega_u g_{\vartheta u2}+g_{\vartheta u1}}{g_{\vartheta u1}}k_\vartheta\gamma\frac{T_E E^{1/2}}{L} \times \frac{1+\langle\Phi\rangle L(g_{u4}+c_2/c_1 g_{\vartheta 4})/(k_\vartheta T_E E^{1/2})}{1+c_2/c_1\langle\Phi\rangle q_{u2}}\right\}.$$

The expression for the turbulent Prandtl number of gas laden with particles follows from the set of equations for the second single-point moments of the carrier phase velocity and temperature fluctuations, with closing relations (23) taken into account

$$Pr_t = \frac{k_\vartheta}{k} \frac{(1+f_{u2}\langle\Phi\rangle)\left[1+\frac{c_{1\vartheta}}{k_\vartheta}\left(1+\frac{1}{Pr_1}\right)\frac{1}{Re_E} + \frac{\langle\Phi\rangle}{k_\vartheta\gamma}\left(g_{u4}+\frac{c_2}{c_1}g_{\vartheta 4}\right)\right]}{\left(1+\frac{c_2}{c_1}\langle\Phi\rangle q_{u2}\right)\left[1+\frac{c_{1E}}{k\,Re_E}+\frac{2f_{u2}\langle\Phi\rangle}{k\gamma}\right]}.$$

For a single-phase turbulent flow ($\langle\Phi\rangle = 0$) it follows from equations (30) that $Pr_t \sim 1+1/Pr_1$ in the near-wall region where $Re_E \ll 1$. Away from the channel walls, the effect of molecular Prandtl number becomes weaker and in the logarithmic region of flow, where $Re_E \gg 1$, $Pr_t \sim k_\vartheta/k$.

The carrier phase temperature fluctuations are calculated on the basis of the equation obtained from equation (21) with the use of approximation relations (23)

$$\langle U_x\rangle\left(1+\frac{c_2}{c_1}\langle\Phi\rangle f_{\vartheta 2}\right)\frac{\partial\langle\vartheta_1^2\rangle}{\partial x} + 2\langle\vartheta_1 u_r\rangle\left(1+\frac{c_2}{c_1}\langle\Phi\rangle q_{\vartheta u2}\right)\frac{\partial\langle\Theta_1\rangle}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}$$
$$\times\left\{r\left[\kappa_1+\left(1+2\frac{c_2}{c_1}\langle\Phi\rangle f_{\vartheta 2}\right)\alpha_{1\vartheta}LE^{1/2}\right]\frac{\partial\langle\Theta_1^2\rangle}{\partial r}\right\}$$
$$-c_\vartheta\frac{E^{1/2}}{L}\langle\vartheta_1^2\rangle\left(1+2\frac{c_2}{c_1}\langle\Phi\rangle\frac{c_E}{c_\vartheta}f_{\vartheta 5}\right)-c_{1\vartheta}\frac{\langle\Theta_1^2\rangle}{L^2}. \quad (25)$$

The solution of the carrier phase energy equation (24) is considered at the given heat flux q_w on the inner surface of the tube. The Nusselt number is found from the formula

$$Nu = \frac{2Rq_w}{\kappa_1(\Theta_w-\Theta_m)} = -\frac{2R}{(\Theta_w-\Theta_m)}\left.\frac{\partial\Theta_1}{\partial r}\right|_{r=R}.$$

The distribution of temperature fluctuations over the tube cross-section is determined from equation (25) provided there are no temperature fluctuations on the tube walls. The values of the constants k, k_ϑ, $c_{1\vartheta}$, c_ϑ are selected the same as for a single-phase flow [12]; the constant γ is determined in ref. [1].

5. CALCULATION RESULTS

As compared with the hydrodynamic problem the analysis of the effect of particles on heat transfer requires that two additional parameters be taken into account: the Prandtl number which is involved in the ratio between thermal and dynamic inertia parameters of particles (thermal and dynamic relaxation times) $\Omega_\vartheta/\Omega = \tau_\vartheta/\tau_u = 3Pr_1/2c_2/c_1$, and the ratio between the particle material and gas heat capacities which enters both in $\Omega_\vartheta/\Omega_u$ and in the 'thermal' concentration $c_2/c_1\langle\Phi\rangle$. The effect of the parameters Pr_1 and c_2/c_1 on the characteristics of turbulent heat transfer is significant for small particles and is negligible for large ones, since the effect of high-inertia particles on the rate of turbulent momentum and heat transfer is insignificant within the framework of the analysis performed. Figure 2 illustrates the dependence of the intensity of gas temperature fluctuations on the Prandtl number and on the ratio of the particle material and gas heat capacities. Particles with the low dynamic inertia parameter $\Omega_u < 1$ enhance the carrier phase temperature fluctuations when the thermal inertia parameter $\Omega_\vartheta = 1$. An increase in the Prandtl number of the carrier phase or in the heat capacity ratio c_2/c_1 leads to a growth of the thermal inertia parameter of the admixture and this reduces the intensity of the gas temperature fluctuations. For inertia particles ($\Omega_u > 1$) a drop in the liquid phase temperature fluctuations is observed, with the maximum of temperature fluctuations being located closer to tube walls.

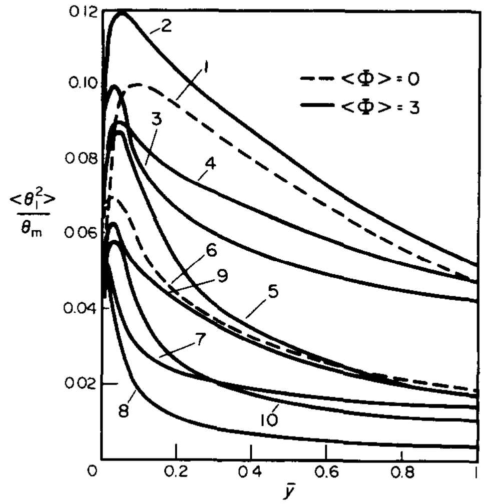

FIG. 2. Effect of the ratio between the thermophysical properties of the particle material and gas on the intensity of gas temperature fluctuations ($Re = 2.2\times10^4$): 1, $\langle\Phi\rangle = 0$, $Pr_1 = 0.7$; 2, $\langle\Phi\rangle = 3$, $Pr_1 = 0.7$, $R/a = 5000$, $c_2/c_1 = 0.5$; 3, $\langle\Phi\rangle = 3$, $Pr_1 = 0.7$, $R/a = 5000$, $c_2/c_1 = 2$; 4, $\langle\Phi\rangle = 3$, $Pr_1 = 0.7$, $R/a = 600$, $c_2/c_1 = 0.5$; 5, $\langle\Phi\rangle = 3$, $Pr_1 = 0.7$, $R/a = 600$, $c_2/c_1 = 2$; 6, $\langle\Phi\rangle = 0$, $Pr_1 = 5$; 7, $\langle\Phi\rangle = 3$, $Pr_1 = 5$, $R/a = 5000$, $c_2/c_1 = 0.5$; 8, $\langle\Phi\rangle = 3$, $Pr_1 = 5$, $R/a = 5000$, $c_2/c_1 = 2$; 9, $\langle\Phi\rangle = 3$, $Pr_1 = 5$, $c_2/c_1 = 0.5$; 10, $\langle\Phi\rangle = 3$, $Pr_1 = 5$, $R/a = 600$, $c_2/c_1 = 2$.

Figure 3 illustrates the effect of the admixture mass concentration on the thermal diffusivity of the carrier phase. It is seen that low inertia particles lead to a monotonous increase in the turbulent thermal diffusivity coefficient of the carrier phase, with the degree of this increase diminishing with the growth of the Prandtl number for the gas or of the ratio between the heat capacities of the particles and gas. For particles with higher inertia $\Omega_u > 1$, the dependence of the turbulent thermal diffusivity coefficient is not monotonous: a drop in the turbulent thermal diffu-

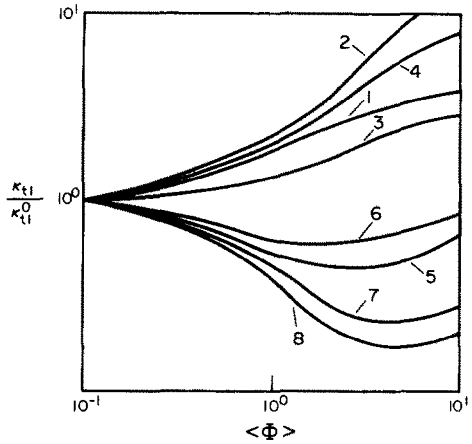

FIG. 3. Effect of the admixture mass concentration on the turbulent thermal diffusivity of gas ($Re = 2.2 \times 10^4$, $\bar{r} = 1$): 1, $R/a = 5000$, $c_2/c_1 = 0.5$, $Pr_1 = 0.7$; 2, $R/a = 5000$, $c_2/c_1 = 2$, $Pr_1 = 0.7$; 3, $R/a = 5000$, $c_2/c_1 = 0.5$, $Pr_1 = 5$; 4, $R/a = 5000$, $c_2/c_1 = 2$, $Pr_1 = 5$; 5, $R/a = 600$, $c_2/c_1 = 0.5$, $Pr_1 = 0.7$; 6, $R/a = 600$, $c_2/c_1 = 2$, $Pr_1 = 0.7$; 7, $R/a = 600$ $c_2/c_1 = 0.5$, $Pr_1 = 5$; 8, $R/a = 600$, $c_2/c_1 = 2$, $Pr_1 = 5$.

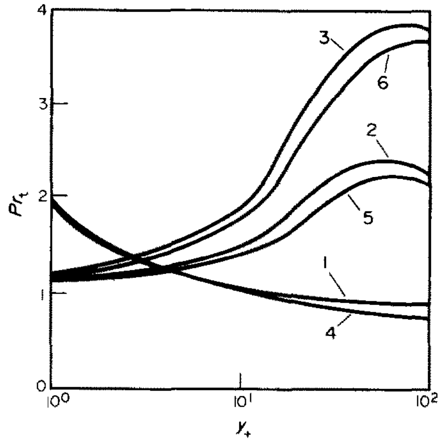

FIG. 5. Distribution of the turbulent Prandtl number of gas laden with particles near the tube wall (for symbols, see Fig. 4).

sivity coefficient of the carrier phase is followed by an increase when the admixture mass concentration changes from 0 to $\langle \Phi \rangle = 3$–5. The higher the value of the particle thermal inertia parameter, the lower are located the curves of $\kappa_{t1}/\kappa_{t1}^0$ vs the particle weight concentration.

The dependence of turbulence on the molecular Prandtl number is more significant in gas suspension flows than in single-phase flows: to greater Pr_1 there correspond greater turbulent Prandtl numbers (Fig. 4). This is explained by the fact that an increase in the Prandtl number of the gas, which causes an increase in the thermal inertia parameter, leads to a more intensive temperature slip of phases, i.e. to an increase in the dissipation of the liquid phase temperature fluctuations. The location of the maximum turbulent Prandtl number of the gas laden with particles in the near-wall region (Fig. 5) is explained by the maximum dissipation of the intensity of temperature fluctuations at $\Omega_9 = 1$.

The ratio between the thermo-physical properties of the particle material and gas exerts an influence not only on the fluctuating temperature structure of the flow, but also on the heat transfer rate of the dusty flow as a whole. Figure 6 shows the dependence of molecular Prandtl number of the carrier phase. An increase in the molecular Prandtl number of the gas entails a decrease in the Nusselt number of the gas suspension flow as compared with the Nusselt number for a single-phase flow. An increase in the heat capacity of the solid phase leads to an increase in heat transfer of the dusty flow with the degree of the gas suspension Nusselt number growth being at maximum when $c_2/c_1 \ll 1$ and decreasing when c_2/c_1

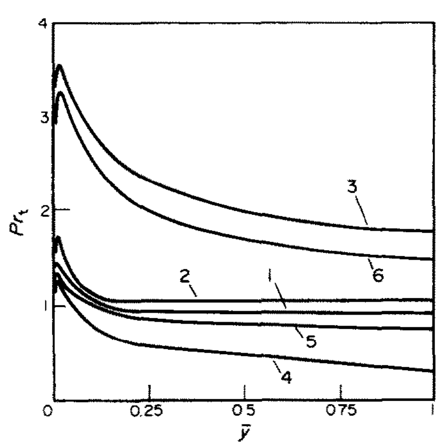

FIG. 4. Distribution of the turbulent Prandtl number of gas over the tube cross-section ($Re = 3 \times 10^4$, $\langle \Phi \rangle = 5$, $R/a = 5000$): 1, $Pr_1 = 0.7$, $c_2/c_1 = 1$; 2, $Pr_1 = 5$, $c_2/c_1 = 1$; 3, $Pr_1 = 20$, $c_2/c_1 = 1$; 4, $Pr_1 = 0.7$, $c_2/c_1 = 4$; 5, $Pr_1 = 5$, $c_2/c_1 = 4$; 6, $Pr_1 = 20$, $c_2/c_1 = 4$.

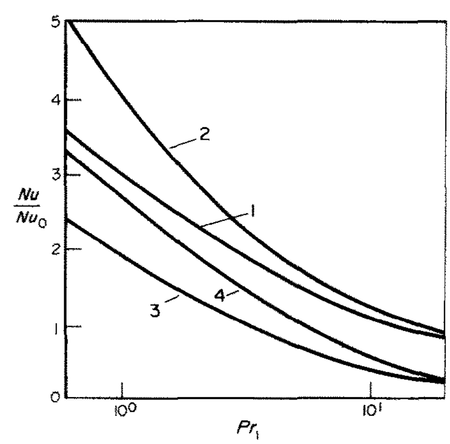

FIG. 6. Effect of the molecular Prandtl number of gas on the Nusselt number of gas suspension ($Re = 3 \times 10^4$, $\langle \Phi \rangle = 5$): 1, $R/a = 5000$, $c_2/c_1 = 1$; 2, $R/a = 5000$, $c_2/c_1 = 4$; 3, $R/a = 3000$, $c_2/c_1 = 1$; 4, $R/a = 3000$, $c_2/c_1 = 4$.

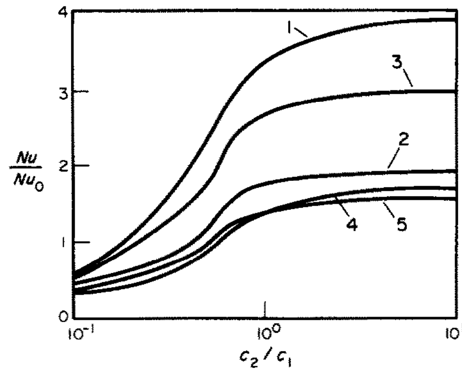

FIG. 7. The Nusselt number of a dusty flow vs the ratio between the heat capacities of the particle material and gas ($Re = 3\times10^4$, $\langle\Phi\rangle = 5$): 1, $Pr_1 = 0.7$, $R/a = 5000$; 2, $Pr_1 = 5$, $R/a = 5000$; 3, $Pr_1 = 0.7$, $R/a = 3000$; 4, $Pr_1 = 5$, $R/a = 2000$; 5, $Pr_1 = 0.7$, $R/a = 2000$.

$\gg 1$ (Fig. 7). For particles with higher inertia, the effect of the ratio between the heat capacities of the particle material and gas on heat transfer diminishes. The relevant results obtained for a dusty flow correlate well with the experimental results of refs. [7, 8].

In Fig. 8 comparison is made between experimental [8] and calculated Nusselt numbers for gas suspension flows in circular tubes. It is seen that the results agree satisfactorily especially for small particles. Note that heat transfer calculations from formulae that neglect the effects of the turbulent flow nonuniformity considerably understates the results as compared with experimental data.

6. CONCLUSIONS

(1) Using the method of averaging over the ensemble of turbulent flow realizations, equations are obtained for averaged heat transfer by a discrete phase and by the gas suspension flow as a whole. The equations show that the entrainment of particles into pulsating motion increases the contribution of particles into turbulent heat transfer.

(2) Closed expressions are obtained for correlations of solid and carrier phase characteristic fluctuations in terms of the second single-point moments of velocity and temperature fluctuations of the carrier phase alone. Taking account of the turbulent flow nonuniformity in the set of equations for the second single-point moments of velocity and temperature fluctuations and the square of temperature fluctuations of the carrier phase with particles leads to the appearance of new terms that describe the generation of convective transfer and turbulent diffusion of the liquid phase temperature fluctuations. The presence of particles in a turbulent flow also leads to an additional dissipation of temperature fluctuations due to the interphase fluctuating temperature slip. The degree of the effect of the discrete admixture on heat transfer depends not only on the mass concentration, relative particle size and the density ratio of the particle material and gas, but also on the ratio between the heat capacities of the particle material and gas and on the carrier phase Prandtl number.

(3) Without using additional constants for the presence of particles in the flow, calculations of the gas suspension flow heat transfer in a circular tube were performed. It is found that, depending on the relative size, density ratio and the ratio between the heat capacities of the particle material and gas, as well as on the gas Prandtl number, both a decrease and an increase in the intensity of turbulent heat transfer processes in a gas suspension are possible. The nonuniformity character of the effect of admixture mass concentration on the turbulent thermal diffusivity coefficient of the carrying gas was determined from calculations. Comparison of the predicted Nusselt numbers of gas suspension with experimental data indicates a satisfactory description of the processes of the turbulent heat transfer in a dusty flow.

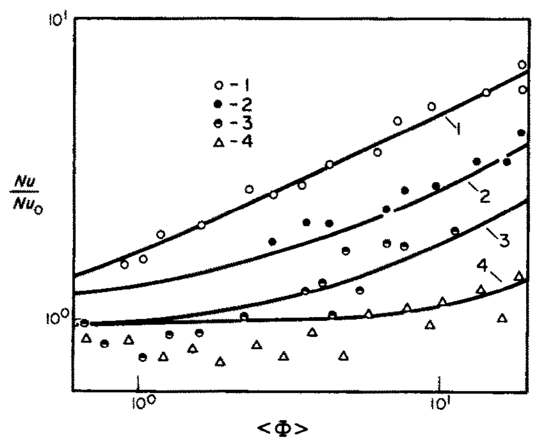

FIG. 8. The Nusselt number for the flow in tubes: 1, $Re = 10^4$, $R/a = 2500$; 2, $Re = 1.2\times10^4$, $R/a = 1100$; 3, $Re = 1.35\times10^4$, $R/a = 600$; 4, $Re = 1.5\times10^4$, $R/a = 200$.

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HYDRODYNAMIQUE ET TRANSFERT THERMIQUE DES ECOULEMENTS TURBULENTS GAZEUX AVEC SUSPENSION DANS DES TUBES—2. TRANSFERT THERMIQUE

Résumé—Par la méthode de moyenne sur l'ensemble des réalisations d'écoulements turbulents, on écrit les équations du transfert de chaleur moyen pour une phase solide et pour le fluide considérés comme un tout. On trouve les expressions des moments de second ordre au même point des fluctuations de vitesse et de température de la phase fluide en présence des particules. Des calculs de transfert de chaleur sont développés pour des écoulements turbulents de suspension dans des tubes circulaires. On étudie les effets de couplage entre les propriétés thermiques et physiques du matériau particulaire et du gaz, sur les caractéristiques thermiques d'un écoulement diphasique. Les nombres de Nusselt calculés pour un gaz poussiereux s'accordent de façon satisfaisante avec les données expérimentales.

HYDRODYNAMIK UND WÄRMETRANSPORT BEI TURBULENTER STRÖMUNG EINER GASSUSPENSION IM ROHR—2. WÄRMETRANSPORT

Zusammenfassung—Durch das Verfahren der Ensemble-Mittelung in einer turbulenten Strömung werden die Mittelwertgleichungen für den Wärmetransport der festen Phase und der Strömung insgesamt abgeleitet. Es werden geschlossene Ausdrücke für das zweite Moment der Geschwindigkeits- und Temperatur-Fluktuationen für Feststoff und Trägergas ermittelt, und zwar in Abhängigkeit vom zweiten Moment der Fluktuationen im Trägergas bei ungleichmäßiger turbulenter Strömung. Auf diesen Ausdrücken aufbauend, wird ein Gleichungssystem für das zweite Moment der Fluktuationen von Geschwindigkeit und Temperatur in Gegenwart von Feststoffteilchen erstellt. Für turbulente Strömung einer Gassuspension in kreisförmigen Rohren wird der Wärmeübergang berechnet. Die Auswirkung des Zusammenhangs zwischen den thermischen und den physikalischen Eigenschaften des Partikelmaterials und des Gases auf die thermischen Eigenschaften einer 2-Phasen-Strömung werden untersucht. Die berechneten Nusselt-Zahlen für eine Staubströmung stimmen zufriedenstellend mit experimentellen Daten überein.

ГИДРОДИНАМИКА И ТЕПЛООБМЕН ПРИ ТУРБУЛЕНТНОМ ТЕЧЕНИИ ГАЗОВЗВЕСИ В ТРУБАХ—2. ТЕПЛООБМЕН

Аннотация—Методом осреднения по ансамблю реализаций турбулентного потока получены уравнения осредненного теплопереноса твердой фазы и потока в целом. Найдены замкнутые выражения для вторых одноточечных моментов пульсаций скорости и температуры твердой и несущей фаз через вторые моменты пульсаций скорости и температуры несущей фазы в неоднородном турбулентном потоке. На основе полученных выражений записана система уравнений для вторых одноточечных моментов пульсаций скорости и температуры жидкой фазы в присутствии частиц. Проведены расчеты теплообмена при турбулентном течении газовзвеси в круглых трубах. Исследовано влияние соотношения теплофизических свойств материала частиц и газа на тепловые характеристики двухфазного потока. Расчетные данные по числу Нуссельта запыленного потока удвлетворительно согласуются с экспериментом.